

# MATHEMATICS (MATH)

## Explanation of Course Numbers

- Courses in the 1000s are primarily introductory undergraduate courses
- Those in the 2000s to 4000s are upper-level undergraduate courses that also may be taken for graduate credit with permission and additional work assigned
- Those in the 6000s and 8000s are for master's, doctoral, and professional-level students
- The 6000s are open to advanced undergraduate students with approval of the instructor and the dean or advising office

Note: MATH 1220 and MATH 1221 each cover one-half the material of MATH 1231. Because MATH 1221, MATH 1231, and MATH 1252 are related in their subject matter, credit for only one of the three may be applied toward a degree. Information on the placement exam, which is an option for placing into MATH 1051, MATH 1220, MATH 1231, or MATH 1252 is at: <http://math.columbian.gwu.edu/gw-mathematics-placement-test>. (<http://math.columbian.gwu.edu/gw-mathematics-placement-test/>)

### **MATH 1000. Dean's Seminar. 3 Credits.**

The Dean's Seminars provide Columbian College first-year students focused scholarship on specific intellectual challenges. Topics vary by semester; see department for more details.

### **MATH 1007. Mathematics and Politics. 3 Credits.**

A mathematical treatment of fair representation, voting systems, power, and conflict; impossibility theorems of Balinski and Young and of Arrow; the electoral college; the prisoner's dilemma.

### **MATH 1008. History of Mathematics. 3 Credits.**

The history of mathematics, with emphasis on its importance in the evolution of human thought. Students learn some useful mathematics from areas such as geometry, number theory, and probability and develop an appreciation of the mathematical endeavor.

### **MATH 1009. Mathematical Ideas I. 3 Credits.**

Elementary mathematical models of growth and decay, scaling, chaos, and fractals.

### **MATH 1051. Finite Mathematics for the Social and Management Sciences. 3 Credits.**

Systems of linear equations, matrix algebra, linear programming, probability theory, and mathematics of finance. Restricted to students with a minimum score of 61 on the ALEKS placement examination.

### **MATH 1099. Variable Topics. 1-36 Credits.**

### **MATH 1220. Calculus with Precalculus I. 3 Credits.**

An introduction to single-variable calculus (differentiation and integration of algebraic and trigonometric functions with applications), with the concepts and techniques of precalculus developed as needed. Prerequisites: students with a minimum score of 61 on the ALEKS placement examination.

### **MATH 1221. Calculus with Precalculus II. 3 Credits.**

Continuation of MATH 1220. An introduction to single-variable calculus (differentiation and integration of algebraic and trigonometric functions with applications), with the concepts and techniques of precalculus developed as needed. Prerequisite: MATH 1220.

### **MATH 1231. Single-Variable Calculus I. 3 Credits.**

Limits and continuity; differentiation and integration of algebraic and trigonometric functions with applications. Restricted to students with a minimum score of 76 on the ALEKS placement examination.

### **MATH 1232. Single-Variable Calculus II. 3 Credits.**

The calculus of exponential and logarithmic functions. L'Hopital's rule. Techniques of integration. Infinite series and Taylor series. Polar coordinates. Prerequisites: MATH 1221 or MATH 1231.

### **MATH 1252. Calculus for the Social and Management Sciences. 3 Credits.**

Differential and integral calculus of functions of one variable; applications to business and economics. Students who might wish to take MATH 1232 should take one of its prerequisites, MATH 1221 or MATH 1231, instead of this course. Prerequisites: A minimum test score of 61 on the ALEKS placement examination.

### **MATH 2000. Sophomore Colloquium. 3 Credits.**

The Sophomore Colloquia are small, seminar-style courses limited to second-year students in Columbian College. These courses engage students deeply in a discipline, focus on a narrow issue of high interest and impact, and require independent research projects of the students. Topics vary by semester. See department for more details.

### **MATH 2184. Linear Algebra I. 3 Credits.**

Linear equations, matrices, inverses, and determinants. Vector spaces, rank, eigenvalues, and diagonalization. Applications to geometry and ordinary differential equations. Credit cannot be earned for both MATH 2184 and MATH 2185. Prerequisites: MATH 1221 or MATH 1231 or MATH 1252; or permission of the instructor.

### **MATH 2185. Comprehensive Introduction to Linear Algebra. 3 Credits.**

Rigorous, proof-based treatment of vector spaces, linear transformations, matrices, determinants, eigenvalues, diagonalization, inner products, and orthogonality, with due regard to computation. Prerequisites: MATH 1221 or MATH 1231; and MATH 2971 or MATH 2971W. Corequisites: MATH 2971 or MATH 2971W may be taken as a corequisite. Credit cannot be earned for this course and MATH 2184.

### **MATH 2233. Multivariable Calculus. 3 Credits.**

Partial derivatives and multiple integrals. Vector-valued functions. Line and surface integrals and the theorems of Gauss, Green, and Stokes. Prerequisites: MATH 1232.

### **MATH 2971. Introduction to Mathematical Reasoning. 3 Credits.**

Introduction to the fundamental abstract concepts of modern mathematics; various proof techniques demonstrated using examples from discrete and continuous mathematics. MATH 1232 may be taken as a corequisite. Prerequisites: MATH 1232 or permission of the department undergraduate advisor.

**MATH 2971W. Introduction to Mathematical Reasoning. 3 Credits.**

Introduction to the fundamental abstract concepts of modern mathematics; various proof techniques demonstrated using examples from discrete and continuous mathematics. Math 1232 may be taken concurrently; permission of instructor or the departmental undergraduate advisor may substituted for the prerequisite. Includes a significant engagement in writing as a form of critical inquiry and scholarly expression to satisfy the WID requirement. Prerequisite: MATH 1232.

**MATH 2991. Introductory Special Topics. 1-3 Credits.**

Permission of the instructor required prior to enrollment. May be repeated for credit.

**MATH 3099. Variable Topics. 1-12 Credits.**

**MATH 3120. Elementary Number Theory. 3 Credits.**

Divisibility of integers, prime numbers, greatest common divisor, the Euclidean algorithm, congruence, the Chinese remainder theorem, number theoretic functions, Möbius inversion, Euler's phi function, and applications to cryptography and primality testing. Prerequisites: MATH 2971 or MATH 2971W.

**MATH 3125. Linear Algebra II. 3 Credits.**

Advanced topics in linear algebra; duality of vector spaces, normal and self-adjoint operators, the singular value decomposition theorem, the spectral theorem, bilinear and quadratic forms, the geometry of orthogonal operators, the Jordan canonical form, and minimal polynomials. Prerequisites: MATH 2971 or MATH 2971W and MATH 2185.

**MATH 3257. Introduction to Complex Variables. 3 Credits.**

Analytic functions and power series; contour integration and the calculus of residues; conformal mapping; physical applications. Prerequisites: MATH 2184 or MATH 2185; MATH 2233; and MATH 2971 or MATH 2971W.

**MATH 3342. Ordinary Differential Equations. 3 Credits.**

A first course in ordinary differential equations, with an emphasis on mathematical modeling: solution curves, direction fields, existence and uniqueness, approximate solutions, first-order and second-order linear equations, linear systems, and phase portraits. Prerequisites: MATH 2233; MATH 2184 or MATH 2185.

**MATH 3343. Partial Differential Equations. 3 Credits.**

A first course in partial differential equations. Fourier series and separation of variables, vibrations of a string, Sturm–Liouville problems, series solutions, Bessel's equation, linear partial differential equations, wave and heat equations. Prerequisites: MATH 3342.

**MATH 3359. Introduction to Mathematical Modeling. 3 Credits.**

Introduction to the fundamental modeling ideas of dimensional analysis, scaling, and elementary approximations of curves and functions; applications to development of models from science and engineering. Prerequisites: CSCI 1011 or CSCI 1012 or CSCI 1041 or CSCI 1111 or CSCI 1121 or CSCI 1131; and MATH 3342.

**MATH 3410. Mathematics of Finance. 3 Credits.**

Mathematical development and analysis of realistic models for financial option pricing; mathematical underpinnings and financial concepts. Prerequisites: MATH 2233.

**MATH 3411. Stochastic Calculus Methods in Finance. 3 Credits.**

Review of probability theory, Brownian motion, Ito integrals, Ito's formula, martingales, stochastic differential equations, boundary value problems, the Dirichlet problem, the Black-Scholes equation, optimal stopping, and American options. Prerequisites: MATH 2184 or MATH 2185; and MATH 3410; or permission of the instructor.

**MATH 3553. Introduction to Numerical Analysis. 3 Credits.**

Accuracy and precision; linear systems and matrices; direct and iterative methods for solution of linear equations; sparse matrices; solution of nonlinear equations. Interpolation and approximate representation of functions, splines. Prerequisites: MATH 2184 or MATH 2185; and MATH 2233; and CSCI 1011 or CSCI 1012 or CSCI 1041 or CSCI 1111 or CSCI 1121 or CSCI 1131.

**MATH 3613. Introduction to Combinatorics. 3 Credits.**

Introduction to combinatorial enumeration; basic counting techniques, inclusion–exclusion principle, recurrence relations, generating functions, pigeonhole principle, bijective correspondences. Prerequisites: MATH 2971 or MATH 2971W.

**MATH 3632. Introduction to Graph Theory. 3 Credits.**

Fundamental concepts, techniques, and results of graph theory; connectivity, traversability, matchings, coverings, colorability, planarity, networks, and Polya enumeration. Prerequisites: MATH 2971 or MATH 2971W.

**MATH 3710. Introduction to Mathematical Logic. 3 Credits.**

Symbolic logic as a precise formalization of deductive thought; logical correctness of reasoning; formal languages, interpretations, and truth; propositional logic and first-order quantifier logic suited to deductions encountered in mathematics; Goedel's completeness theorem; compactness. Prerequisites: MATH 2971 or MATH 2971W.

**MATH 3720. Axiomatic Set Theory. 3 Credits.**

Cantor's theory of sets. Russell's paradox. Axiomatization of set theory as a framework for a contradiction-free mathematics. The Zermelo–Fraenkel axioms and the axiom of choice. Finite, countable, and uncountable sets; ordinal and cardinal arithmetic. The continuum hypothesis. Prerequisites: MATH 2971 or MATH 2971W or permission of instructor.

**MATH 3730. Computability Theory. 3 Credits.**

The unlimited register machine as a model of an idealized computer. Computable and partial computable functions; Church–Turing thesis. Kleene's recursion theorem. Algorithmic enumerability. Unsolvability of the halting problem and other theoretical limitations on what computers can do. Discussion of Goedel's incompleteness theorem. Prerequisites: MATH 2971 or MATH 2971W or permission of instructor.

**MATH 3740. Computational Complexity. 3 Credits.**

Automata and languages; deterministic and nondeterministic Turing machines; space and time complexity measures and classes; P-versus-NP problem; traveling salesman problem and other NP-complete problems; intractability; circuit complexity; introduction to probabilistic and quantum algorithms. Prerequisites: MATH 2971 or MATH 2971W.

**MATH 3806. Introduction to Topology. 3 Credits.**

Metric spaces: completeness, compactness, continuity: Topological spaces: continuity, bases, subbases, separation axioms, compactness, local compactness, connectedness, product and quotient spaces. Prerequisites: MATH 2971 or MATH 2971W.

**MATH 3848. Differential Geometry. 3 Credits.**

Curves in space, regular surfaces, tensors, fundamental forms of a surface, Gauss–Bonnet theory, minimal surfaces; the geometry of the Gauss map. Prerequisites: MATH 2184 or MATH 2185; MATH 2233; and MATH 2971 or MATH 2971W.

**MATH 4121. Introduction to Abstract Algebra I. 3 Credits.**

Study of groups and associated concepts, including Lagrange’s theorem, Cayley’s theorem, the fundamental theorem of homomorphisms, and applications to counting. Prerequisites: MATH 2184 or MATH 2185; and MATH 2971 or MATH 2971W.

**MATH 4122. Introduction to Abstract Algebra II. 3 Credits.**

Study of rings, through maximal and prime ideals, and the study of fields, through Galois theory. Prerequisites: MATH 4121 or permission of the instructor.

**MATH 4239. Real Analysis I. 3 Credits.**

Rigorous study of differentiation, integration, and convergence; sequences and series, continuity and differentiability of real-valued functions of a real variable, the Riemann integral, sequences of functions, and power series. Prerequisites: MATH 1232 and MATH 2971 or MATH 2971W or permission of instructor.

**MATH 4239W. Real Analysis I. 3 Credits.**

A rigorous study of differentiation, integration, and convergence. Topics include sequences and series, continuity and differentiability of real-valued functions of a real variable, the Riemann integral, sequences of functions, and power series. Includes a significant engagement in writing as a form of critical inquiry and scholarly expression to satisfy the WID requirement. Prerequisites: MATH 1232; and MATH 2971 or MATH 2971W.

**MATH 4240. Real Analysis II. 3 Credits.**

Continuation of MATH 4239. Topology of  $n$ -dimensional space, derivatives of functions of several variables, inverse and implicit function theorems, multiple integrals, generalized Stokes’s theorem. Prerequisites: MATH 2184 or MATH 2185; and MATH 2233; and MATH 4239 or MATH 4239W. Credit cannot be earned for this course and MATH 6202.

**MATH 4981. Seminar: Topics in Mathematics. 3 Credits.**

Topics vary by semester. May be repeated for credit provided the topic differs. See department for more details. Prerequisites: MATH 2184 or MATH 2185; and MATH 2233.

**MATH 4991. Special Topics. 1-12 Credits.**

Permission of the instructor required prior to enrollment. May be repeated for credit.

**MATH 4995. Reading and Research. 1-6 Credits.**

Under the personal direction of an instructor. Limited to majors with demonstrated capability. May be repeated for credit. Restricted to permission from instructor.

**MATH 4995W. Reading and Research. 1-6 Credits.**

Intended exclusively for advanced mathematics majors writing a senior thesis. The thesis is the required outcome of the course. Prerequisites: Permission of the instructor. Includes a significant engagement in writing as a form of critical inquiry and scholarly expression to satisfy the WID requirement.

**MATH 5099. Variable Topics. 1-99 Credits.****MATH 6101. Algebra I. 3 Credits.**

Group theory including symmetric groups, free abelian groups, finitely generated abelian groups, group actions, Sylow theorems, solvable groups. Ring theory including factorization in commutative rings, rings of polynomials, chain conditions.

**MATH 6102. Algebra II. 3 Credits.**

Continuation of MATH 6101. Theory of modules, including modules over a principal ideal domain and tensor product of modules. Theory of fields, including finite fields and Galois theory.

**MATH 6120. Topics in Algebra. 3 Credits.**

Topics may include, but are not limited to, Lie groups and Lie algebras, non-associative algebras, abelian groups, classical groups, algebraic number theory, representation theory, algebraic geometry, and ring theory. May be repeated for credit with permission. Prerequisites: MATH 6101 and MATH 6102.

**MATH 6201. Real Analysis I. 3 Credits.**

A rigorous study of the real number system, metric spaces, topological spaces, product topology, convergence, continuity and differentiation. Topics include Dedekind’s cuts, Tychonoff’s theorem, sequences and series, Abel’s theorem, continuity and differentiability of real-valued functions of a real variable. Credit may not be earned for both MATH 6201 and MATH 4239.

**MATH 6202. Real Analysis II. 3 Credits.**

Continuation of MATH 6201. Topics include Riemann–Stieltjes integrals, equicontinuity, Arzela–Ascoli theorem, Stone–Weierstrass theorem, derivatives of functions of several variables, contraction mapping theorem, inverse and implicit function theorems, differential forms, exterior differentiation, Stokes’s theorem, differentiable manifolds. Credit may not be earned for both MATH 6202 and MATH 4240.

**MATH 6214. Measure and Integration Theory. 3 Credits.**

Lebesgue measure and integration in abstract spaces. Probability measures. Absolute continuity, the Radon–Nikodym theorem, measures on product spaces, and the Fubini theorem.  $L^p$  spaces and their properties. Prerequisite: MATH 4239.

**MATH 6215. Introduction to Functional Analysis. 3 Credits.**

Topological and metric spaces; Tychonoff theorem; Banach spaces; linear functionals and operators; Hahn–Banach, closed graph, and open-mapping theorems; uniform boundedness; Hilbert spaces; eigenvalues, projections. Prerequisite: MATH 6214 .

**MATH 6225. Ergodic Theory. 3 Credits.**

Ergodicity, mixing, the K-property and the Bernoulli property. Poincaré recurrence, the Rohlin lemma, the ergodic theorem, and entropy theory. Additional topics from isomorphism theory, spectral theory, the theory of joinings, and coding theory. Prerequisites: MATH 6214 or permission of the instructor.

**MATH 6226. Dynamical Systems and Chaos. 3 Credits.**

Linear and nonlinear systems, flows, Poincaré maps, structural stability. Examples of chaotic systems in the physical sciences. Local bifurcations, center manifold theory, normal forms, the averaging theorem. Hyperbolic invariant sets, strange attractors, the Smale horseshoe, symbolic dynamics. Prerequisites: MATH 2184 and MATH 4240; or permission of the instructor.

**MATH 6230. Complex Analysis. 3 Credits.**

Topology of the complex plane; complex differentiation and integration; Cauchy's theorem and its consequences; Taylor and Laurent series; classification of singularities; residue theory; conformal mapping; the Riemann mapping theorem. Prerequisite: MATH 4239.

**MATH 6240. Topics in Real and Functional Analysis. 3 Credits.**

Possible topics include Banach algebras, function algebras, spectral theory for bounded and unbounded operators, harmonic analysis on topological groups and semigroups, topological vector spaces and operator algebras. May be repeated for credit with permission. Prerequisites: Permission of the instructor.

**MATH 6318. Applied Mathematics I. 3 Credits.**

Boundary value problems in one dimension, first order equations, method of characteristics, shock waves, linear elliptic and evolution equations, calculus of variations. In addition to the specified prerequisites, students must have completed an undergraduate course in differential equations prior to enrollment. Prerequisites: MATH 2184 and Math 2233.

**MATH 6319. Applied Mathematics II. 3 Credits.**

Stability and bifurcation, perturbation methods, Sobolev spaces, wave equation, nonlinear partial differential equations. Students must have taken an undergraduate course in real analysis in addition to the specified prerequisites. Prerequisites: MATH 2184 and Math 2233.

**MATH 6330. Ordinary Differential Equations. 3 Credits.**

Existence and uniqueness of solutions, continuity and differentiability of solutions with respect to initial conditions. Properties of linear systems, phase portraits, planar systems and Poincaré–Bendixson theory. Prerequisite: MATH 4240.

**MATH 6340. Modern Partial Differential Equations. 3 Credits.**

Emphasis on modern theory and analytical techniques applied to the solution of partial differential equations. Topics include Sobolev spaces, generalized solutions, strong solutions and regularity; Sobolev imbedding theorem; Rellich–Kondrachov theorem; Leray–Schauder fixed-point theorems; nonlinear eigenvalue problems. Prerequisites: MATH 6319 or permission of the instructor.

**MATH 6350. Topics in Applied Mathematics. 3 Credits.**

Possible topics include, but are not limited to, the calculus of variations, control theory, nonlinear partial differential equations, and mathematical programming. May be repeated for credit with permission.

**MATH 6441. Introduction to Financial Mathematics. 3 Credits.**

Elementary finance. Basic probability. Discrete random variables. Forwards, futures, and options. Options and arbitrage. The binomial model. Cox–Ross–Rubenstein formula. Martingales. Continuous random variables. The continuous model as a limit of the binomial model. Prerequisites: MATH 2184 and MATH 2233.

**MATH 6442. Stochastic Calculus Methods in Finance. 3 Credits.**

Review of finance and probability theory. Brownian motion. Ito's formula. Martingales. Stochastic differential equations. The Black–Scholes equation. Optimal stopping. American options. Prerequisites: MATH 2184 and MATH 2233.

**MATH 6522. Introduction to Numerical Analysis. 3 Credits.**

Computer arithmetic and round-off errors. Solution of linear and nonlinear systems. Interpolation and approximations. Numerical differentiation and integration. Eigenvalues and eigenvectors. Prerequisite: MATH 1232 and MATH 2184 and knowledge of a programming language.

**MATH 6523. Numerical Solution of Ordinary and Partial Differential Equations. 3 Credits.**

Initial and boundary value problems for ordinary differential equations. Error propagation, convergence and stability. Finite difference and finite element methods for partial differential equations. Prerequisite: MATH 3342 and knowledge of a programming language.

**MATH 6540. Topics in Numerical Analysis. 3 Credits.**

Numerical methods and software. Introductions to the methods, tools, and ideas of numerical computation. Problem solving using standard mathematical software. Interpolation; linear and nonlinear equations. Differential equations. Prerequisites: MATH 3342 and knowledge of a programming language.

**MATH 6555. Numerical Linear Algebra and Introduction to Python. 3 Credits.**

Introduction to numerical methods for linear algebra. Topics might include direct methods for solving linear systems, iterative methods for solving the linear systems, eigenvalue problems; least squares problems, and optimization problems. Prerequisites: MATH 2184.

**MATH 6575. Deep Learning with Applications to Deterministic and Stochastic Models. 3 Credits.**

This course is aimed at advanced undergraduate and beginning graduate students in STEM. This course provides a broad entry point to applied data science for students who know linear algebra, differential equations, scientific computing. Prerequisites: Math 2184 Linear Algebra I Math 2233 Multi-variable Calculus.

**MATH 6610. Combinatorics. 3 Credits.**

An introduction to fundamental methods and current research problems in partially ordered sets and enumeration. Prerequisites: Undergraduate modern algebra and linear algebra or permission of the instructor.

**MATH 6620. Graph Theory. 3 Credits.**

Graphical enumeration, factors, planarity and graph coloring, algebraic graph theory, extremal graph theory, applications. Prerequisites: Undergraduate modern algebra and linear algebra or permission of the instructor.

**MATH 6630. Topics in Combinatorial Mathematics. 3 Credits.**

Topics selected from a wide range of research subjects in combinatorics, its relations with other areas of mathematics, and applications. Recent selections have included matroid theory, topological methods in ordered sets, algebraic methods in combinatorics, fractional graph theory, combinatorics of polytopes, the symmetric group. May be repeated for credit with permission.

**MATH 6710. Mathematical Logic. 3 Credits.**

Model theory: the relation between a formal language (syntax) and its interpretations (semantics). Consistency, completeness, and compactness. Tarski's theorem on the inexpressibility of truth. Godel's incompleteness theorem and its impact on mathematics.

**MATH 6720. Topics in Logic. 3 Credits.**

Topics selected from a broad spectrum of areas of logic and applications, based on students' suggestions and interests. Recent selections have included computable mathematics, computable model theory, computability theory, set theory, and algorithmic learning theory. May be repeated for credit with permission.

**MATH 6810. General Topology. 3 Credits.**

Topological spaces, bases and subbases, open sets and closed sets; continuous maps and homeomorphisms; connectedness and compactness; metric topology, product topology, and quotient topology; separation axioms; finite topological spaces, covering spaces, and fundamental groups.

**MATH 6820. Algebraic Topology. 3 Credits.**

Fundamental groups and the Van Kampen theorem; simplicial complexes, simplicial homology, and Euler characteristic; singular homology, Mayer-Vietoris sequences. Topics may include cohomology, cup products, and Poincaré duality; classification of surfaces; knots and their fundamental groups. Prerequisites: MATH 6810 or permission of the instructor.

**MATH 6850. Knot Theory and Low Dimensional Topology. 3 Credits.**

Introduction to fundamental methods and current research in knot theory and 3-dimensional topology. Topics include Reidemeister moves, Alexander invariants, Jones-type invariants, skein modules, Khovanov homology, incompressible surfaces, and torus decomposition. Prerequisites: MATH 6810 or permission of the instructor.

**MATH 6860. Topics in Knot Theory and Low Dimensional Topology. 3 Credits.**

Possible topics include, but are not limited to, topology of 3-manifolds and work of Perelman, quantum invariants and their categorizations, topology of 4-manifolds after Freedman and Donaldson, computational complexity in topology, and applications in biology, chemistry, and physics. May be repeated for credit with permission. Prerequisites: MATH 6850 or permission of the instructor.

**MATH 6890. Topics in Topology. 3 Credits.**

Topics may include hyperbolic structures on surfaces and 3-manifolds; knot theory; topology of 3-manifolds; topology of 4-manifolds. Prerequisite: MATH 6820 or permission of the instructor. May be repeated for credit with permission.

**MATH 6991. Graduate Student Experience. 0 Credits.**

Introduction to the experience of studying mathematics as a graduate student at GW. Understanding University rules and regulations, conducting research and delivering presentations, and pursuing a successful career as a mathematician. Restricted to graduate students in the department.

**MATH 6995. Reading and Research. 12 Credits.**

May be repeated for credit.

**MATH 8999. Dissertation Research. 3-12 Credits.**

May be repeated for credit. Restricted to doctoral candidates.